The Physics and Mechanics of Liquefaction
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Abstract
Simple physics and mechanics are used to explain the first order shape, structure and
distribution of various surface ejecta as well as sub-surface manifestation of liquefaction. We
begin with simple mechanical and fluid models, and discuss how changing parameters such as
depth of water table, type of soil, and pre-existing liquefaction dykes will affect the
characteristics of liquefaction. We then provide examples of field observations of the simple
model by dissecting the observations from Canterbury Earthquake Sequence and extracting
quantifiable features. We find how shear strength and thickness of top soil cap affects the
surface manifestation and rate of dissipation of pore pressure.

1. Introduction

Fig. 1: Flowchart showing factors that affect liquefaction susceptibility.
Liquefaction is the transformation of a saturated granular material from a solid state to a fluid state as a consequence of increased pore-water pressure (Youd, 1973; Seed and Idriss, 1982). Liquefaction is caused by the application of shear stresses, which causes soil particles to lose contact with each other, and a buildup of interstitial pore-water pressure. The liquefied mixture of sand and water acts as a viscous liquid, which can rise up through dykes.

The systematic study of liquefaction is a young discipline. Accordingly, some of the physical parameters that control liquefaction effects in the field are not completely understood (Obermeier, 2009). While civil and geotechnical engineers have attempted to study shear strength and liquefaction susceptibility by empirical, experimental and theoretical methods, few studies integrate geologic studies and field observations. We attempt to bridge this gap.

A variety of approaches have been used to study liquefaction. Liu and Qiao (1984) have reported shaking table experiments on stratified sand to understand sand boil generation during earthquakes, both in the free field and around structures. Other experimental techniques include centrifuge studies, tri-shear tests and cyclic loading studies to find out failure thresholds for different ratios of silt and sand in a mixture. Quigley et al. (2013) recorded and mapped liquefaction in Christchurch during 2010-2011 earthquakes as the liquefaction was taking place. Tuttle (2001) did extensive mapping of liquefied areas in Bhuj, India in the days following a M 7.9 earthquake, and recorded in detail the diameter and height of sand volcanoes. Green et al. (2004) have mapped preserved paleo-liquefaction features to study paleoseismicity.

Theoretical techniques like energy calculations have been utilized by Okur and Ansal (2011). Green and Terry (2005) have used Palmgren–Miner P–M cumulative damage hypothesis developed for soil fatigue evaluations. Popescu R. (2002) and many others have used 1-D, 2-D and 3-D finite element models and determined outputs for varying input parameters like frequency and amplitude of shaking and void ratio. Current literature abounds in statistical analysis of empirical data. Haldar (1979) and Tang devised a probabilistic approach, as opposed
to deterministic. Castilla and Audemard (2007) have used sand blows as an empirical tool for magnitude estimation of pre-instrumental earthquakes by using best fit functions.

Our application of basic mechanics to understanding liquefaction is a first of its kind. Our simple models give interesting, previously unexplored insights. For example, we explore how physical properties of non-liquefiable top soil can influence the rate of dissipation of pore fluid pressures at depth. Poulos et al. (1985) state that liquefaction can also occur in very large masses of sands or silts that are dry and loose and loaded so rapidly that the escape of air from the voids is restricted. Such movement of dry and loose sands is often referred to as running soil or running ground. Although such soil may flow as liquefied soil does, in this paper we only consider the liquefaction that occurs for soils that are located below the groundwater table.

![Flowchart showing types of features found at liquefaction sites.](image)

**2. Condition for surface manifestation of liquefaction**

Field observations show pathways of preferential release of liquefied material and often reactivation of old features. Within this section we offer an explanation for why nature behaves in this way.

Threshold for soil cap rupture depends on the cap shear strength and thickness and force exerted by rising fluid in the dyke. Soil derives its shear strength from two sources:
1. Cohesion between particles (stress independent component):
   Cementation between sand grains
   Electrostatic attraction between clay particles

2. Frictional resistance between particles (stress dependent component):
   Angularity

Fig. 3: Mohr-Coulomb Failure Criterion shows C as cohesion or tensile strength and \( \mu = \tan \Phi \) as friction.

Failure takes place when \( \tau > C + \sigma_n \cdot \tan \Phi \)

\[
\tau = \frac{P}{A} \quad (1)
\]

Where \( P \) is the resultant shearing force through the centroid of the area \( A \) being sheared. \( \sigma_n \) is the normal stress, which in this case is overburden pressure. \( A \) is area of the cylinder sheared out of the soil cap:

\[
A = 2 \pi r H \quad (2)
\]
Where \( r \) = radius of dike and \( H \) = thickness of soil cap. Since overburden or vertical pressure is the direction of \( \sigma_1 \), and \( \sigma_2 \) and \( \sigma_3 \) are confining pressures, \( \sigma_n = \sigma_1 \).

Thus the criterion for soil cap rupture is:

\[
P > (C + \sigma_n \mu) 2 \pi r H
\]  

(3)

This implies that **the thicker** is the soil cap, more fluid pressure is required to break it. This is the condition for a homogeneous soil layer, with no pre-existing zones of weakness.

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Fig. 4: Shows a case where (a) dike or (b) dikes have been unable to breach the top layer. Though Liquefaction took place, it has not been manifested at the surface. This will have a significant impact on hazard planning and zoning, since the areas that might not have seemed to have liquefied, might have lost structural integrity at depth, near the foundations of houses and buildings.
We now find out what pressure is needed to re-activate a pre-existing dike.

\[
\tau = \mu \ast \sigma_n
\]  

(4)

Note that

\[
\sigma_n = \sigma_1 \cos \theta + \sigma_2 \sin \theta
\]  

(5)

where \(\theta\) is the angle with horizontal.

Since dykes are sub-vertical, \(\cos \Theta\) will tend to zero and \(\sigma_n \approx \sigma_2\). By convention, \(\sigma_1 > \sigma_2\).

Physically, overburden pressure stress is \(\sigma_1\) and confining stress is \(\sigma_2\).

\[
\tau > \mu \ast \sigma_2
\]  

(6)

\[
P_{reactivating} > \mu \ast \sigma_2 \ast 2 \pi r H
\]  

(7)

Now comparing the pressure needed to create a new dike versus pressure needed to reactivate a dike,

\[
\frac{P_{reactivation}}{P_{new\ dike}} = \frac{\mu \ast \sigma_2}{C + \mu \ast \sigma_1} < 1
\]  

(8)
Thus, it is often observed that younger dikes reactivate along the boundary that is a zone of weakness in older dikes. Similarly, small tubular dikes have been seen to develop in pre-existing holes left behind by decayed roots and in holes excavated by creatures such as crabs or crawfish (Audemard and de Santis, 1991). In Quigley et. al. (2013) field study, a one area of the field showed no evidence for sand blow deposition after the 4 September 2010 Darfield earthquake. Then, a small (~10 cm diameter) hole was cored to depths of ~2 m, where the liquefiable layer was encountered. This hole gradually closed at the surface prior to the Christchurch earthquake, but erupted as a source conduit for sand blow formation in the 22 February, 13 June, and 23 December 2011 earthquakes. They noted that preexisting zones of weakness in the near surface (e.g., higher-permeability fracture zones or conduits through otherwise low-permeability layers overlying the liquefiable layer) exert a first-order control on the vent distribution of sand blows by providing more-efficient pathways for liquefied material to move vertically.

2.1. Field examples
As shown above, the minimum pore pressure required to rupture the top soil cap is proportional to the radius of the dike. This implies that it is easier for a thinner dike to make it to the surface, as opposed to a dike with larger radius, as seen in Fig. 6.

\[
P_{\text{min}} = (C + \sigma_n * \mu) * 2 \pi r H
\]
Fig. 6 (a) Is photograph from Kaiapoi (near Waimakariri river, north of Christchurch), and (b) from Sullivan Park at Avonside, shows a thinner dike breaching more of top soil. The thicker dike terminates at a deeper depth, and in Fig. 6(a), the liquefied material spreads as a sill, and feeds into the thinner dike.

2.2. Exceptions and Anomalies:

Our simple criterion for reactivation proposed above is insufficient to explain some complex behaviour. We qualitatively explain some conflicting observations in this section. Fig. 7 is a case where the contact between a paleo-dike and the surrounding soil has had enough time to cement in the vadose zone. Water was retained by a combination of adhesion (funicular groundwater), and capillary action in the already fluid rich dike material that remained it the dike after upwelling stopped. Fig. 7 (b) shows how this caused vadose silt to cement the particles in the dike and increase the shear strength of the dike. Hence, the youngest dike, instead of breaching the older contact or older dikes, found a new path, and cross-cut them to make a vertical path towards the surface.
Fig. 7: (a) Photograph from Quigley et al. (2013), indicates three distinguishable events. The oxidised and bioturbated nature of the outermost dike indicates this is a paleo-liquefaction feature formed in a pre-CES earthquake. (b) shows development of vadose silt and cementation of loose soil through time.

In the calculation for dike reactivation, the path was assumed sub vertical. But if the dike is at an angle $\Theta$ to the vertical, then:

$$\sigma_n = \sigma_1 \cos \theta + \sigma_2 \sin \theta$$  \hspace{1cm} (11)

Normal stress will be more than just the confining pressure. i.e. $\sigma_n \neq \sigma_z$.

$$\frac{P_{\text{reactivat-vertical dike}}}{P_{\text{reactivat-inclined dike}}} = \frac{\mu \sigma_2}{\mu (\sigma_1 \cos \theta + \sigma_2 \sin \theta)} < 1$$  \hspace{1cm} (12)

This simply implies that it can be more difficult to reactivate an inclined dike, than to reactivate or create a vertical dike.
If the shear strength of the infilled dike material from a previous event is very low due to lack of time to compact and cement, such that it is less than the force required to overcome friction at the walls, then a younger dike will rise between the older fill, rather than the edge of it. Fig. 8 from a trench in Sullivan Park at Avonside, Christchurch shows such a case. Dike A has cut through the middle of an older dike. Also notice a thin dike B has gravel and small pebbles. The gravel source layer was deeper than the silt and sand layers that were mostly liquefied. But why a deeper, coarser liquefied material has risen through a narrow dike: as opposed to ones with larger radius is discussed in the next section.

Fig. 8: From Sullivan Park, Christchurch shows that younger Dike A has cut through the middle of an older dike.

2.3. Implications

Crosscutting relationships of liquefaction dikes can be used to study paleoseismicity, especially where dating techniques do not work. Whether a dike has cut through the middle of a pre-existing dike or the edge of it can be an indication of time between two events. This understanding can improve studies that use liquefaction features to estimate recurrence intervals from strong earthquakes.
3. **Flow through liquefaction dyke as pipe flow**

Pipe flow is a type of liquid flow within a closed conduit. The other type of flow within a conduit is open channel flow. These two types of flow are similar in many ways, but differ in one important aspect. Pipe flow does not have a free surface which is found in open-channel flow. For example, storm sewers are closed conduits but usually maintain a free surface and therefore are considered open-channel flow. The exception to this is when a storm sewer operates at full capacity, and then can become pipe flow.

![Flow profile](image)

Liquefaction dykes will be considered a case of closed conduit pipe flow, since the hydraulic head is above ground level, where the groundwater is being rejected. A case in which a dike is modelled as two parallel planes gives similar results, but only the case of a pipe is discussed here.

Pipe flow is governed by:

1. **The principle** of continuity/conservation of mass:
   
   For an incompressible fluid, this would imply that the rate of inflow = rate of outflow

2. Conservation of energy, i.e. Bernoulli’s equation *(reference, year)*:

   \[
   \frac{p_1}{\rho \cdot g} + z_1 + \frac{v_1^2}{2 \cdot g} = \frac{p_2}{\rho \cdot g} + z_2 + \frac{v_2^2}{2 \cdot g} \tag{13}
   \]

   Here \( p_1 - p_2 \) = pore fluid pressure \( P \) and \( z_1 - z_2 = L \) is depth of liquefied layer.

3. Poiseuille’s equation *(reference, year)* to account for viscosity: Fluid in contact with either surface is held to that surface by adhesive forces between the molecules of the fluid and surface. Therefore, the molecules at the surface of the stationary wall are at
rest and the molecules at the surface of the moving plate will be moving with velocity $v$.
The stationary layer of fluid in contact with the stationary wall will retard the flow of the
layer just above it.

$$\frac{dV}{dt} = Q = \frac{P \pi r^4}{8 \eta L}$$  \hspace{1cm} (14)

4. The Darcy–Weisbach equation (Manning, 1997) relates the pressure loss due
to friction along a given length of pipe to the average velocity of the fluid flow.

$$\Delta p_{friction} = \frac{f L \rho V^2}{4 r} = \frac{16 L \rho v_m^2}{r Re}$$  \hspace{1cm} (15)

The Darcy friction factor for laminar flow (Reynolds number less than 2100) is given by
the following formula:

$$f = \frac{64}{Re} \text{ and } Re = \frac{2 \rho v_m r}{\eta}$$  \hspace{1cm} (16), (17)

Therefore,

$$\Delta p_f = \frac{8 \eta L v_m}{r^2}$$  \hspace{1cm} (18)

Where
\begin{align*}
\eta &= \text{viscosity} \\
Q &= \text{volumetric flow rate} \\
V &= \text{Volume of ejecta} \\
\Delta p &= \text{difference in pressure} \\
\Delta pf &= \text{pressure loss due to friction} \\
L &= \text{length of pipe} \\
D &= \text{diameter of the pipe} = 2r \\
\rho &= \text{density of the fluid} \\
g &= \text{gravitational constant} \\
v_m &= \text{mean velocity of the flow}
\end{align*}
Re = Reynolds number

Expressing the volume output per unit time as a function of velocity:

\[
\frac{dV}{dt} = \pi r^2 \cdot v_m
\]  \hspace{1cm} (19)

Combining this with Poiseuille's equation and friction formula yields:

\[
\frac{dV}{dt} = \left( P - \frac{8 \eta L \cdot v_m}{r^2} \right) \cdot \frac{\pi r^4}{8 \eta L} = \frac{P \cdot \pi r^4}{8 \eta L} - \frac{dV}{dt}
\]  \hspace{1cm} (20)

and

\[
\frac{dV}{dt} = \frac{P \cdot \pi r^4}{16 \eta L}
\]  \hspace{1cm} (21)

3.1. Rate of removal of water with time

Solving this differential equation will give the rate of ejecta volume with time, the total volume of liquefied fluid released as well as the amount of time taken for liquefaction to end and pore pressure to come down to “normal” level or to be less than the confining pressure.

Remember, P is not a constant; it is pore fluid pressure that gradually decreases after shaking has stopped.

So to solve the differential equation, one of the following pieces of information is needed:

1. Pore fluid pressure with time after shaking has ceased. This can be obtained theoretically by calculating the rate of settling due to gravity or empirically by finding a best fit equation for the piezometer data, \( P = (?) f(t) \).

2. Finding the decrease in pore pressure due to the loss of volume, i.e.

\[
P = (?) function \ of \ \int_0^t \frac{dV}{dt}
\]
To find this function, soil can be considered to be a reservoir of a given water volume. Both porosity and permeability would affect this function. Obtaining either of these functions is presently beyond the scope of this paper.

3.2. Implications

Qualitative analysis of the equation gives some interesting insights. For example, increasing angularity of the soil particles increases the viscosity $\eta$ and causes increased loss of pressure head, decreasing the time and volume of surface ejection. Another inference one can make is, smaller radius gives higher velocity but since it increases the sediment carrying capacity of the fluid, as seen in Fig. 7 where a thin dike carried gravel from deeper depth. Volume of ejecta per unit time is proportional to fourth power of radius of dike. This implies that pore fluid pressure will decay much faster if radius of dike is larger, and radius of dike depends on shear strength of soil cap, as shown in Section 3. Hence thickness of cap soil, though non-liquefiable, can affect the dynamics of liquefaction even after the cap is breached. There is a huge scope to further develop this study and compare it with observations.

3.3. Assumptions and approximations:

Viscous fluids that obey Poiseuille’s equation are called Newtonian fluids and $\eta$ is constant independent of the speed of flow. When $\eta$ does depend upon the velocity of flow the fluids are called non-Newtonian. Liquefied sand/ silt and water would be non-Newtonian since it contains sand or silt particles that can deform and become preferentially oriented so that the viscosity decreases to maintain the flow rate. We have also assumed that the flow is laminar, but in fact, it might be turbulent. Turbulent flows are much more difficult to study. The assumptions are listed in mathematical form below:

a. The fluid is incompressible ($dp/dt = 0$).
b. The flow is steady.
c. The flow is in the z-direction only ($v_r = v_\theta = 0$).
d. The flow is axisymmetric ($\frac{\partial v_z}{\partial \theta} = 0$).
3.4. Morphology of a dike:

Friction at the edge of the dike causes a velocity gradient across the pipe, so the velocity is maximum in the middle of the pipe.

\[ v = v_{max} \left( 1 - \frac{r^2}{R^2} \right) \]  

(22)

The slower fluid deposits finer particles such as silt at the walls of the dike as it cannot carry the load, and friction makes it adhere to the walls. Thus, as with time, as the pore fluid pressure drops, the velocity of the fluid in center decreases as well, until it can no longer rise, and deposits the remaining load, which has a higher percentage of coarse sediment such as sand at the center of the dike.

Fig. 10: (a) silt drape at the walls of a dike that is filled with sand in the middle. (b) Pipe Flow with parabolic velocity profile (from Gramoll, ecourses.com)
4. Soil is a memory keeper

Another possible approach to understanding pore pressure is to take a broad view beyond a single event. We can study liquefaction by dealing with data statistically. The goal is to find systems that robustly reproduce the general patterns of volume of ejecta regardless of the details of the local soil microphysics. This an approach of using the past as an indicator of the future, while accounting for factors such as foreshocks and the time gap between two events that can make a difference in the intensity of liquefaction.

Several investigators have noted that liquefaction resistance of soils increases with age. For example, Seed (1979) observed significant increases in liquefaction resistance with aging of reconstituted sand specimens tested in the laboratory. Increases of as much as 25% in cyclic resistance ratio were noted between freshly constituted and 100-day-old specimens. Youd and Hoose (1977) and Youd and Perkins (1978) noted that liquefaction resistance increases markedly with geologic age.

Sediments deposited within the past few thousand years are generally much more susceptible to liquefaction than older Holocene sediments and pre-Pleistocene sediments are generally immune to liquefaction. Although qualitative time-dependent increases have been documented, no quantitative age factor studies have been done.

Fig. 11: Idealized porosity versus depth curves for different lithologies.

\[ \phi = \phi_0 e^{-CZ} \]

\( \phi_0 \) = porosity of top soil
\( \phi_n \) = porosity being calculated for each iteration of burial
\( T_n \) = thickness of the layer
\( T_{n-1} \) = thickness before the last iteration of compaction > \( T_n \)
\( Z \) = depth of burial at each time increment
We use a back stripping formula widely used in basin analysis for measuring compaction through time. Though the scales are largely different, and various conditions can affect the compaction of top soil, this approach is reasonable for first order approximation. Assuming compaction is the only way in which accommodation space is being created, then

$$\phi_n = \phi_o * e^{-cz}$$  \hspace{1cm} (23)

$$T_n = \frac{(1 - \phi_{n-1})}{1 - \phi_n} * T_{n-1}$$ \hspace{1cm} (24)

$$AccSp = \Delta z = T_n - T_{n-1}$$ \hspace{1cm} (25)

This shows that as the top layer gets older, it gets compacted and buried. It is less porous and less susceptible to liquefaction. But this compaction does not create an equivalent amount of accommodation space for younger soil to deposit. With each iteration, the thickness of liquefaction susceptible soil decreases, as the whole column is experiencing an overall densification and decrease of void ratio with time. In addition to the compaction mentioned above, if there is a liquefaction event in the midst of this, it would lead to further settling, consider equivalent of quicker aging.

![Fig. 12: (a) Initial depositional packing, with large voids, (b) Packing during liquefaction, with excess pore-water pressure, (c) Resedimentation, with layer of water at top: compaction.](image)

Above calculation shows that liquefaction resistance increases with time, but on a shorter time scale, liquefaction potential increases. The following Piezometer data obtained from Tonkin and Taylor shows how pore pressure remain elevated for a few minutes after the foreshock, and the starting point of for the following earthquake is higher than normal level.
Fig. 13: Pore Pressure with time at two sites in Christchurch indicates effect of a foreshock on increasing liquefaction potential.

Fig. 14: Piezometer data at Parenga Place shows how tides influence pore pressure.

Note that the y-axis range is different than in Fig. 13. Parenga Place is nearer to sea than other sites. Pore fluid pressures remain elevated after the first shock, and the second shock’s starting fluid pressure is already high, hence the elevated pressure is higher than what it would’ve been for an isolated event of that magnitude.
Future Work:

There exist a plethora of directions this project can branch into and develop further. Field data can be collected in a more rigorous and detailed format than has been available so far. For example, if soil cap thickness varies across a group of trenches, radius of dikes can be measured and plotted against thickness to verify or refute the relationship hypothesized by our simple model.

Piezometer data values can be obtained by writing to Christchurch GeoDatabase, and can be imported into software that gives the equation of the best fit curve. Pore pressure dissipation with time might be an exponential decay, and the decay constant obtained in each case can be compared to radius of dike, to see whether there is any correlation. As hypothesized in Section 4, there might also exist a relationship between thickness of non-liquefiable cap and rate of decay of pore pressure beneath that layer.

In addition to these field and empirical studies, the current models can be refined with models that make fewer assumptions. For example, the liquefied material is assumed to be a viscous fluid, while it is actually neither fluid nor solid, nor wet sand. Elastoplasticity of an intermediate media: a matrix of fluid with high percentage of sediment load, such that the solid particles also partake in a certain percentage of interactions, is yet to be studied. The existing literature on brittle-ductile transition zone in lower crust may give a general idea about how to approach constituting the behavior of such boundary media. Sand volcanoes have a very low angle of repose, mostly ranging between 4° and 10°, as opposed to wet sand castles one would make on a beach, which are as steep as 30°. This low angle of repose can be explored by understanding the elastoplasticity of the intermediate medium discussed above.

One can also explore a less recognized fact that liquefaction like effects can also be triggered by explosion-induced ground motions. Ground motions caused by explosives can produce localized peak accelerations that can be several orders of magnitude greater than earthquake accelerations. Unfortunately, little information is available on the performance and safety of slopes, earthfill dams, levees, canals and other earth structures subjected to vibrations generated by construction blasting.
Finally, though a long way off from any liquefaction discussed above, the skills, insight and acumen obtained in these analyses can help one understand the fundamentals of liquefaction and upwelling of mud diapers in an accretionary wedge if one wishes to work on such projects later in one’s career.

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References:


